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RELIABILITY MEASURE OF FINITE MIXTURE OF RAYLEIGH DISTRIBUTIONS

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ABSTRACT
In reliability, a life time distribution can be characterized by the reliability function, hazard rate function. These functions provide probabilistic information on the residual life time and also aging properties. The residual lifetime tends to decrease, with increasing age of the component. In this paper various measures hazard rate, mean time to failure, variance of the time to failure are derived for finite mixture of Rayleigh distribution.

INTRODUCTION
Finite mixture of distribution provide an important tool in modeling a wide range of observed phenomena, which do not normally yield to modeling through classical distributions like Normal, Gamma, Poisson, Binomial , etc, on account of their heterogeneous nature and inherent complexity. In a finite mixture model, the distribution of random quantity of interst is modeled as a mixture of a finite number of component distributions in varying proportions. A mixture model is thus able to model quite complex situations through an appropriate choice of its components to represents accurately the local areas of support of the true distribution.

In reliability theory, there are lots of real life situations where the concept of mixture distributions can be applied. In this paper various measures hazard rate, mean time to failure, variance of the time to failure are derived for finite mixture of Rayleigh distribution.

STATISTICAL MODEL
A finite mixture of Rayleigh distribution with k- components can be represented in the form

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \ \dots..+p_k f_k(x)$$

With densities $$f_i(x) = \lambda_i x exp\left(-\lambda_i \frac{x^2}{2}\right), \lambda_i > 0, x > 0, p_i > 0, i = 1,2, \dots k{:}\ \sum_{i=1}^{k} p_i = 1$$

HAZARD RATE
Let t denotes life time of a component with the probability density function $f(t)$ is

$$f(t) = p_1\lambda_1 t exp\left(-\lambda_1 \frac{t^2}{2}\right) + p_2\lambda_2 t exp\left(-\lambda_2 \frac{t^2}{2}\right),\ \ \lambda_1, \lambda_2 > 0, t > 0, p_1 + p_2 = 1$$

Then for the models the hazard rate h(t) is given by

$$h(t) = \frac{f(t)}{1 - F(t)}$$

Where $F(t) = \int_0^t f(t)dt = \int_0^t p_1\lambda_1 t exp\left(-\lambda_1 \frac{t^2}{2}\right) + p_2\lambda_2 t exp\left(-\lambda_2 \frac{t^2}{2}\right) dt$

$$= p_1(1 - exp\left(-\lambda_1 \frac{t^2}{2}\right) + p_2\left(1 - exp\left(-\lambda_2 \frac{t^2}{2}\right)\right)$$

$$\therefore h(t) = \frac{p_1\lambda_1 t exp\left(-\lambda_1 \frac{t^2}{2}\right) + p_2\lambda_2 t exp\left(-\lambda_2 \frac{t^2}{2}\right)}{p_1 exp\left(-\lambda_1 \frac{t^2}{2}\right) + p_2 exp\left(-\lambda_2 \frac{t^2}{2}\right)}$$

In general

$$h(t)=\frac{\sum_{i=1}^{k}p_i\lambda_i texp\left(-\lambda_i\frac{t^2}{2}\right)}{\sum_{i=1}^{k}p_i exp\left(-\lambda_i\frac{t^2}{2}\right)}$$

Then cumulative hazard rate function is

$$H(t)=\int_0^t h(x)dx$$

$$=\int_0^t\frac{p_1\lambda_1 xexp\left(-\lambda_1\frac{x^2}{2}\right)+p_2\lambda_2 xexp\left(-\lambda_2\frac{x^2}{2}\right)}{p_1exp\left(-\lambda_1\frac{x^2}{2}\right)+p_2exp\left(-\lambda_2\frac{x^2}{2}\right)}$$

$$H(t)=-\log\left(p_1exp\left(-\lambda_1\frac{t^2}{2}\right)+p_2exp\left(-\lambda_2\frac{t^2}{2}\right)\right)$$

$$\therefore R(t)=e^{-H(t)}=p_1exp\left(-\lambda_1\frac{t^2}{2}\right)+p_2exp\left(-\lambda_2\frac{t^2}{2}\right)$$

In general,

$$H(t)=-log\left(\sum_{i=}^{k}p_iexp\left(-\lambda_i\frac{x^2}{2}\right)\right)$$

$$R(t)=\sum_{i=}^{k}p_iexp\left(-\lambda_i\frac{x^2}{2}\right)$$

$$=\ p_1\lambda_1exp\left(-\lambda_1\frac{x^2}{2}\right)+p_2\lambda_2exp\left(-\lambda_2\frac{x^2}{2}\right)+\ ---+\ p_k\lambda_kexp\left(-\lambda_k\frac{x^2}{2}\right)$$

Mean time to failure (MTTF)

Mean time to failure is the expected time to failure of a component or system is given by

$$MTTF=E(t)=\int_0^\infty tf(t)dt\ or\ \int_0^\infty R(t)dt$$

For k= 2,

$$MTTF=\int_0^\infty\left(p_1exp\left(-\lambda_1\frac{x^2}{2}\right)+p_2texp\left(-\lambda_2\frac{x^2}{2}\right)\right)dt$$

$$MTTF=p_1\sqrt{\frac{\pi}{2\lambda_1}}+p_2\sqrt{\frac{\pi}{2\lambda_2}}=\sqrt{\frac{\pi}{2}}\left(\frac{p_1}{\lambda_1}+\frac{p_2}{\lambda_2}\right)$$

For k=3,

$$MTTF=\sqrt{\frac{\pi}{2}}\left(\frac{p_1}{\lambda_1}+\frac{p_2}{\lambda_2}+\frac{p_3}{\lambda_3}\right)$$

In general,

$$MTTF=\sqrt{\frac{\pi}{2}}\sum_{i=}^{k}\frac{p_i}{\lambda_i}$$

VARIANCE OF THE TIME TO FAILURE

The variance of the time to failure is given by

$$Var(t) = \int_0^\infty t^2 f(t)dt - E^2(t)$$

For k=2,

$$Var(t) = \int_0^\infty t^2 \left(p_1\lambda_1 texp\left(-\lambda_1 \frac{t^2}{2}\right) + p_2\lambda_2 texp\left(-\lambda_2 \frac{t^2}{2}\right) dt\right) - \left(\sqrt{\frac{\pi}{2}}\left(\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2}\right)\right)^2$$

$$= p_1\lambda_1 \frac{2}{\lambda_1{}^2} + p_2\lambda_2 \frac{2}{\lambda_2{}^2} - \frac{\pi}{2}\left(\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2}\right)^2$$

$$Var(t) = 2\left(\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2}\right) - \frac{\pi}{2}\left(\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2}\right)^2$$

For k=3,

$$Var(t) = 2\left(\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2} + \frac{p_3}{\lambda_3}\right) - \frac{\pi}{2}\left(\frac{p_1}{\lambda_1} + \frac{p_2}{\lambda_2} + \frac{p_3}{\lambda_3}\right)^2$$

In general,

$$Var(t) = 2\left(\sum_{i=1}^{k} \frac{p_i}{\lambda_i}\right) - \frac{\pi}{2}\left(\sum_{i=1}^{k} \frac{p_i}{\lambda_i}\right)^2$$

CONCLUSION

Reliability measures hazard rate, mean time to failure, variance time to failure of finite mixture of Rayleigh distributions are derived.

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